# Sensitivity Analysis of the Empirical Model

The following is a sensitivity analysis for the empirical model for propeller noise taken from Chapter 12 of the textbook “Elements of Aviation Acoustics” by G.J.J Ruijgrok. The propeller noise equation is given as:

|  |  |
| --- | --- |
|  |  |
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|  | |

# Input Identification of Regression Analysis Based Sound Models

Following table identifies the different inputs to the several flyover sound models delineated in the technical study “The Development of a Flyover Noise Prediction Technique Using Multiple Linear Regression Analysis” by Cessna Aircraft Co.

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Model Number:** | | **1** | **2** | **3** | **4** | **5** | **7** | **8** | **9** | **10** |
| **Parameter** | **Unit** |  |  |  |  |  |  |  |  |  |
| Helical Mach | - | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Shaft HP | Hp |  |  |  |  |  | ✓ | ✓ | ✓ | ✓ |
| Helical Mach Squared | - |  |  |  |  |  | ✓ |  |  |  |
| Two or Three Blades ? | - |  |  | ✓ |  |  | ✓ | ✓ |  | ✓ |
| Turbocharger Enabled? | - |  |  |  |  |  | ✓ | ✓ |  | ✓ |
| Single or Twin engine? | - |  |  |  |  |  | ✓ |  |  | ✓ |

## Comparison between Models 7 and 10











## Extended Limits Comparison Between Models 7 and 10

Following two figures show an extended limits test for the two models applied to the two relevant inputs i.e. Helical Tip Mach and Shaft Horse Power. Increasing both input sets to twice their original sizes has produced the following figures and it clearly shows that not much of an appreciable difference can be spotted even if the input values are increased past their expected values.





The answer to the question of whether model 7 or model 10 is better, has been answered by the paper itself. Judging from all statistical modelling parameters used to check the validity of all models mentioned in the paper, Model 10 is the best model.

# Codes for both Models

## Empirical Model

clc;clear;close all

%For this code, let us use the data from Chapter 14 examples of Gudmundson

%The model has been taken from Ref 18 - "Elements of aviation Acoustics"

%Starting with the piston engine noise:

n = 2600;%engine rotational speed in RPM

N = 4;%number of cylinders

f\_c = n/120;%cylinder firing frequency

f\_e = (N\*n)/120;%exhaust firing frequency

% According to Reference 61, the overall A-weighted level of the exhaust noise

% of an unmuffled piston engine at 150 m sideline can be estimated by:

P\_br = 310\*745.7;%engine shaft power, watts

L\_A = 8 + (14\*log10(P\_br)) %dBA

%Using data in Gudmundson - An airplane is powered by a two-bladed propeller

%whose diameter is 76 inches is driven by a 310 BHP engine

%Then moving on to propeller noise:

% For the prediction of far-field propeller noise, the following expres sion for the

% maximum sound pressure level can be used:

B = 3;%number of blades per propeller

n\_p = 2600;%propeller rotational speed (rpm)

D = 1.9304;%propeller diameter (m)

r = 50;%distance from propeller (m)

c = 340;%speed of sound(m/s)

M\_t = (pi\*D\*n\_p)/(60\*c);%tip mach number

SPL\_max = 83.4 + (15.3\*log10(P\_br)) - (20\*log10(D)) + (38.5\*M\_t) + ...

(-3\*(B - 2)) + (10\*log10(N)) - (20\*log10(r))

%Sensitivity Analysis for the propeller model

%First we will vary each input parameter indvidually to check its impact on

%the sound level while keeping the other inputs constant.

%Sensitivity to Number of Blades

figure(1)

B\_set = [2, 3, 4, 5, 6];

SPL\_max\_B = 83.4 + (15.3.\*log10(P\_br)) - (20.\*log10(D)) + (38.5.\*M\_t) + ...

(-3.\*(B\_set - 2)) + (10.\*log10(N)) - (20.\*log10(r))

plot(B\_set,SPL\_max\_B,'r','LineWidth',2);grid on

xlabel('Number of blades');ylabel('SPL\_{max} (dBA)');title('SPL variation with No. of Blades')

%Sensitivity to Engine Horse Power

figure(2)

P\_br\_set = [200:10:400].\*745.7;%watts

SPL\_max\_P\_br = 83.4 + (15.3.\*log10(P\_br\_set)) - (20.\*log10(D)) + (38.5.\*M\_t) + ...

(-3.\*(B - 2)) + (10.\*log10(N)) - (20.\*log10(r))

plot(P\_br\_set,SPL\_max\_P\_br,'g','LineWidth',2);grid on

xlabel('engine shaft power (watts)');ylabel('SPL\_{max} (dBA)');title('SPL variation with engine shaft power')

%Sensitivity to Propeller RPM i.e. tip mach

figure(3)

n\_p\_set = 1000:100:3000

M\_t\_set = (pi.\*D.\*n\_p\_set)./(60.\*c);%tip mach number

SPL\_max\_n\_p = 83.4 + (15.3.\*log10(P\_br)) - (20.\*log10(D)) + (38.5.\*M\_t\_set) + ...

(-3.\*(B - 2)) + (10.\*log10(N)) - (20.\*log10(r))

plot(n\_p\_set,SPL\_max\_n\_p,'b','LineWidth',2);grid on

xlabel('Propeller RPM');ylabel('SPL\_{max} (dBA)');title('SPL variation with propeller RPM')

%Sensitivity to Propeller Diameter i.e. also affects tip mach

figure(4)

D\_set = 0.5:0.1:2;%propeller diameter (m)

M\_t\_set\_2 = (pi.\*D\_set.\*n\_p)./(60.\*c);%tip mach number

SPL\_max\_D = 83.4 + (15.3.\*log10(P\_br)) - (20.\*log10(D\_set)) + (38.5.\*M\_t\_set\_2) + ...

(-3.\*(B - 2)) + (10.\*log10(N)) - (20.\*log10(r))

plot(D\_set,SPL\_max\_D,'c','LineWidth',2);grid on

xlabel('Propeller Diameter (m)');ylabel('SPL\_{max} (dBA)');title('SPL variation with propeller diameter')

%Sensitivity to distance From Propeller

figure(5)

r\_set = 0:10:100;%distance from propeller (m)

SPL\_max\_r = 83.4 + (15.3.\*log10(P\_br)) - (20.\*log10(D)) + (38.5.\*M\_t) + ...

(-3.\*(B - 2)) + (10.\*log10(N)) - (20.\*log10(r\_set))

plot(r\_set,SPL\_max\_r,'k','LineWidth',2);grid on

xlabel('distance from propeller (m)');ylabel('SPL\_{max} (dBA)');title('SPL variation with distance from propeller')

## Regression Models

%Following are the regression based models from the paper:

% "The Development of a Flyover Noise Prediction Technique Using Multiple

% Linear Regression Analysis"

clc;clear variables;close all

%Defining the State Conditions at different altitudes

%specifying all of the altitudes

alt\_ft = 1000:1000:35000;

alt\_m = alt\_ft \* 0.3048;

%Temp in K, Sound Speed in m/s, Pressure in Pa, rho(h) in kg/m3

[T\_si,a\_si,P\_si,rho\_si] = atmoscoesa(alt\_m);

%converting atmospheric values to english uni`ts

T\_eng = 1.8\*T\_si;%rankine

a = 3.28084\*a\_si;%ft/s

P = 0.02088547\*P\_si;%lb/ft^2

rho = 0.00194032\*rho\_si;%slugs/ft^3

%MODEL # 1 - (Single Engine Aircraft)

X\_1 = 0.79;%Helical Mach

Y\_1 = 12.7506 + (75.6219\*X\_1);%dBA

%MODEL # 2 - (Twin Engine Aircraft)

X\_2 = 0.78;%Helical Mach

Y\_2 = 25.99 + (65.0586\*X\_2);%dBA

%MODEL # 3 - (Single and Twin Engine)

X\_3\_1 = 0.78;%Helical Mach

X\_3\_2 = 0;% 0 for single and 1 for double prop

Y\_3 = 13.2314 + (75.0445\*X\_3\_1) + (4.3295\*X\_3\_2);%dBA

%MODEL # 4 - (Single Engine Aircraft)

X\_4 = 0.78;%Helical Mach

Y\_4 = 86.7697 + (137.8972\*X\_4);%dBA

%MODEL # 5 - (Single Engine Aircraft)

X\_5 = 0.78;%Helical Mach

Y\_5 = 60.8837 + (0.00481\*X\_5);%dBA

%MODEL # 6 - ILLEGIBLE

%MODEL # 8 - (Single Engine Aircraft)

X\_8\_1 = 250;%BHP

X\_8\_2 = 0.78;%Helical Mach

X\_8\_3 = 0;%0 for 2 blade, 1 for 3 blade

X\_8\_4 = 0;%0 for non-turbo, 1 for turbo

Y\_8 = 30.5646 + (0.00942\*X\_8\_1) + (49.9636\*X\_8\_2) + (2.4494\*X\_8\_3) + (0.4552\*X\_8\_4);

%MODEL # 9 - (Twin Engine Aircraft)

X\_9\_1 = 250;%BHP

X\_9\_2 = 0.78;%Helical Mach

Y\_9 = 5.2566 + (0.01428\*X\_9\_1) + (84.2969\*X\_9\_2);

%MODEL # 10 -

X\_10\_1 = 250;%BHP

X\_10\_2 = 0.78;%Helical Mach

X\_10\_3 = 0;%0 for 2 blade, 1 for 3 blade

X\_10\_4 = 0;%0 for non-turbo, 1 for turbo

X\_10\_5 = 0;%0 for single engine, 1 for twin engine

Y\_10 = 28.8194 + (0.00678\*X\_10\_1) + (52.6543\*X\_10\_2) + (2.8333\*X\_10\_3)...

+ (0.2603\*X\_10\_4) + (2.5742\*X\_10\_5);

%MODEL # 7 - (Sing and Twin Engine Aircraft)

X\_7\_1 = 250;%BHP

X\_7\_2 = 0.78;%Helical Mach

X\_7\_3 = X\_7\_2^2;%Helical Mach Squared

X\_7\_4 = 0;%0 for 2 blade, 1 for 3 blade

X\_7\_5 = 0;%0 for non-turbo, 1 for turbo

X\_7\_6 = 0;%0 for single engine, 1 for twin engine

Y\_7 = 31.3920 + (0.0067\*X\_7\_1) + (46.1576\*X\_7\_2) + (4.2376\*X\_7\_3) + ...

(2.5981\*X\_7\_4) + (0.2577\*X\_7\_5) + (2.6106\*X\_7\_6);

%COMPARISONS BETWEEN MODELS 7 & 10

%Helical Tip Mmach Variance Comparison:

%Single engine (250 hp), 2 blade, non turbo

figure(1)

M\_hel\_Set = 0.2:0.05:0.8;

Y\_7\_Mach\_1 = 31.3920 + (0.0067.\*250) + (46.1576.\*M\_hel\_Set) + (4.2376.\*(M\_hel\_Set.^2)) + ...

(2.5981.\*0) + (0.2577.\*0) + (2.6106.\*0);

Y\_10\_Mach\_1 = 28.8194 + (0.00678\*250) + (52.6543\*M\_hel\_Set) + (2.8333\*0)...

+ (0.2603\*0) + (2.5742\*0);

%Twin Engine, 3 blade, turbo

Y\_7\_Mach\_2 = 31.3920 + (0.0067.\*250) + (46.1576.\*M\_hel\_Set) + (4.2376.\*(M\_hel\_Set.^2)) + ...

(2.5981.\*1) + (0.2577.\*1) + (2.6106.\*1);

Y\_10\_Mach\_2 = 28.8194 + (0.00678\*250) + (52.6543\*M\_hel\_Set) + (2.8333\*1)...

+ (0.2603\*1) + (2.5742\*1);

plot(M\_hel\_Set,Y\_7\_Mach\_1,'r','LineWidth',2);grid on;hold on

plot(M\_hel\_Set,Y\_10\_Mach\_1,'g','LineWidth',2)

plot(M\_hel\_Set,Y\_7\_Mach\_2,'b','LineWidth',2);

plot(M\_hel\_Set,Y\_10\_Mach\_2,'k','LineWidth',2)

xlabel('Helical Tip Mach');ylabel('SPL\_{max} (dBA)');title('comparison for varying mach numbers')

legend('Model 7 (Single engine, 2 blade, non turbo)', 'Model 10 (Single engine, 2 blade, non turbo)',...

'Model 7 (Twin Engine, 3 blade, turbo)', 'Model 10 (Twin Engine, 3 blade, turbo)','Location','NorthWest')

xlim([0.2 0.8])

%Shaft Horse Power Variance Comparison:

%Mach 0.78, 2 blade, non turbo

figure(2)

M\_hel\_Set = 0.78;

Power\_Set = 100:10:500;%hp

Y\_7\_Power\_1 = 31.3920 + (0.0067.\*Power\_Set) + (46.1576.\*M\_hel\_Set) + (4.2376.\*(M\_hel\_Set.^2)) + ...

(2.5981.\*0) + (0.2577.\*0) + (2.6106.\*0);

Y\_10\_Power\_1 = 28.8194 + (0.00678.\*Power\_Set) + (52.6543.\*M\_hel\_Set) + (2.8333.\*0)...

+ (0.2603.\*0) + (2.5742.\*0);

%Twin Engine, 3 blade, turbo

Y\_7\_Power\_2 = 31.3920 + (0.0067.\*Power\_Set) + (46.1576.\*M\_hel\_Set) + (4.2376.\*(M\_hel\_Set.^2)) + ...

(2.5981.\*1) + (0.2577.\*1) + (2.6106.\*1);

Y\_10\_Power\_2 = 28.8194 + (0.00678\*Power\_Set) + (52.6543\*M\_hel\_Set) + (2.8333\*1)...

+ (0.2603\*1) + (2.5742\*1);

plot(Power\_Set,Y\_7\_Power\_1,'r','LineWidth',2);grid on;hold on

plot(Power\_Set,Y\_10\_Power\_1,'g','LineWidth',2)

plot(Power\_Set,Y\_7\_Power\_2,'b','LineWidth',2);

plot(Power\_Set,Y\_10\_Power\_2,'k','LineWidth',2)

xlabel('Shaft Power (hp)');ylabel('SPL\_{max} (dBA)');title('comparison for varying shaft powers')

legend('Model 7 (Single engine, 2 blade, non turbo)', 'Model 10 (Single engine, 2 blade, non turbo)',...

'Model 7 (Twin Engine, 3 blade, turbo)', 'Model 10 (Twin Engine, 3 blade, turbo)','Location','NorthWest')

%Setting up comparisons via models 7 and 10 w.r.t Diameter, RPM, and Speed

%Assuming configuration through all comparisons to be -> 250 hp, 2 blade, non turbo

%forward velocity of 50 ft/s if unvaried, Diameter of 6.33333 ft if unvaried

%Forward Speed:

figure(3)

alt = 1;

R = 6.3333/2; %ft

RPM = 2600;%rpm

RPM\_rads = RPM\*0.1047198;%rad/s

V\_forward = 50:10:250;%ft/s

V\_hel\_2 = sqrt((V\_forward.^2) + ((RPM\_rads.\*R).^2));

M\_hel\_Set\_2 = V\_hel\_2./a(alt)

Y\_7\_Forward = 31.3920 + (0.0067.\*250) + (46.1576.\*M\_hel\_Set\_2) + (4.2376.\*(M\_hel\_Set\_2.^2)) + ...

(2.5981.\*0) + (0.2577.\*0) + (2.6106.\*0);

Y\_10\_Forward = 28.8194 + (0.00678\*250) + (52.6543\*M\_hel\_Set\_2) + (2.8333\*0)...

+ (0.2603\*0) + (2.5742\*0);

plot(V\_forward,Y\_7\_Forward,'c','LineWidth',2);grid on;hold on

plot(V\_forward,Y\_10\_Forward,'m','LineWidth',2)

legend('Model 7 (Single engine, 2 blade, non turbo)',...

'Model 10 (Single engine, 2 blade, non turbo)')

xlabel('Shaft Power (hp)');ylabel('SPL\_{max} (dBA)');title('comparison for varying Forward Speeds')

%Prop Diameter

figure(4)

R\_set = (2:1:7)./2; %ft

RPM = 2600;%rpm

RPM\_rads = RPM\*0.1047198;%rad/s

V\_forward = 50;%ft/s

V\_hel\_3 = sqrt((V\_forward.^2) + ((RPM\_rads.\*R\_set).^2));

M\_hel\_Set\_3 = V\_hel\_3./a(alt)

Y\_7\_Rad = 31.3920 + (0.0067.\*250) + (46.1576.\*M\_hel\_Set\_3) + (4.2376.\*(M\_hel\_Set\_3.^2)) + ...

(2.5981.\*0) + (0.2577.\*0) + (2.6106.\*0);

Y\_10\_Rad = 28.8194 + (0.00678\*250) + (52.6543\*M\_hel\_Set\_3) + (2.8333\*0)...

+ (0.2603\*0) + (2.5742\*0);

plot(R\_set,Y\_7\_Rad,'r','LineWidth',2);grid on;hold on

plot(R\_set,Y\_10\_Rad,'g','LineWidth',2)

legend('Model 7 (Single engine, 2 blade, non turbo)',...

'Model 10 (Single engine, 2 blade, non turbo)')

xlabel('Prop Radii');ylabel('SPL\_{max} (dBA)');title('comparison for varying Prop Radii')

%Propeller RPM

figure(5)

R\_set = 6.33333/2; %ft

RPM\_set = 1000:100:2700;%rpm

RPM\_rads = RPM\_set.\*0.1047198;%rad/s

V\_forward = 50;%ft/s

V\_hel\_4 = sqrt((V\_forward.^2) + ((RPM\_rads.\*R\_set).^2));

M\_hel\_Set\_4 = V\_hel\_4./a(alt)

Y\_7\_RPM = 31.3920 + (0.0067.\*250) + (46.1576.\*M\_hel\_Set\_4) + (4.2376.\*(M\_hel\_Set\_4.^2)) + ...

(2.5981.\*0) + (0.2577.\*0) + (2.6106.\*0);

Y\_10\_RPM = 28.8194 + (0.00678\*250) + (52.6543\*M\_hel\_Set\_4) + (2.8333\*0)...

+ (0.2603\*0) + (2.5742\*0);

plot(RPM\_set,Y\_7\_RPM,'b','LineWidth',2);grid on;hold on

plot(RPM\_set,Y\_10\_RPM,'k','LineWidth',2)

legend('Model 7 (Single engine, 2 blade, non turbo)',...

'Model 10 (Single engine, 2 blade, non turbo)')

xlabel('RPM');ylabel('SPL\_{max} (dBA)');title('comparison for varying RPMs')